On the strong-coupling effects in jet bremsstrahlung in the heavy flavor decays

Nikolai Uraltsev[*]

INFN, Sezione di Milano, Milano, Italy
and
Department of Physics, University of Notre Dame du Lac, Notre Dame, IN 46556, USA

Abstract

We address the strong-coupling regime effects in the light-like quark jets where the radiated gluons are hard but highly collinear. These may lead to additional contributions in the invariant mass (or recoil energy) spectra, on top of the Fermi motion of the initial heavy quark in decays like $b \to s + \gamma$. It is shown that the integer moments are nevertheless free from such effects; perturbation theory corrections for the moments are driven by α_s in the small coupling regime. They are modified nonperturbatively by the soft modes but not by the collinear modes.

[*]On leave of absence from St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia

arXiv:hep-ph/0407359v1 30 Jul 2004

1 Introduction

The OPE provides a consistent approach to describe sufficiently inclusive distributions in the decays of heavy flavor hadrons. It incorporates power-suppressed effects which originate from the nonperturbative domain that is responsible for confinement and bound-state dynamics. Its important ingredient is the separation between the contributions of different momentum scales, with the short distances generically referred to the coefficient functions, and large distances $\sim \Lambda_{\rm QCD}^{-1}$ belonging to the nonperturbative operators. This general approach applies to both the weak transitions into sufficiently heavy quarks like $b \to c\, \ell \nu$, and to heavy-to-light decays like $b \to s + \gamma$ or $b \to u\, \ell \nu$. The former case has been best elaborated including the technical implementation of the Wilsonian 'hard' momentum scale separation in the perturbative expansion. This approach appeared successful in describing experimental data [1]. There remain, however certain principal differences between the decays into heavy and light-like quarks when gluon bremsstrahlung is carefully incorporated.

The standard application of the OPE is initially formulated in the Euclidean theory. It utilizes the expansion of the amplitudes in the momentum of the soft quantum fields which describe nonperturbative dynamics. Therefore, it assumes all the components of the momentum to be bounded by some mass μ identified with the normalization scale of the nonperturbative operators. It is normally chosen to be

$$\mu \simeq \ {\rm few} \times \mu_{\rm hadr} \,,$$

where $\mu_{\rm hadr} \simeq 600 \, {\rm MeV}$ represents the typical momentum scale characteristic for the strong coupling domain in QCD. Consequently, for the gluon exchanges those with any component of the momentum k_α larger than μ are attributed to the coefficient functions and are calculated in practice in perturbation theory. The same idea is transferred with minimal modifications to decays in the Minkowski kinematics.

This procedure works fine for the transitions between heavy quarks, in particular in $b \to c\, \ell \nu$. The lower cutoff on the gluon energy automatically eliminates the infrared domain and no running coupling $\alpha_s(q^2)$ enters at the scales $q^2 \ll \mu^2$. The perturbative expansion is therefore applicable and justified.

The situation looks different in the heavy-to-light decays, in particular in $b \to s + \gamma$. The gluon bremsstrahlung is described by the double logarithmic probability

$$dW = \int \frac{d\omega}{\omega} \, \int \frac{dk_\perp^2}{k_\perp^2} \, C_F \frac{\alpha_s(k_\perp^2)}{\pi} \, dW_{\rm born} \,, \tag{1}$$

where $dW_{\rm born}$ is the bare 'hard' decay rate and $k_\mu = (\omega, k_\perp, k_\parallel)$ is the gluon four-momentum. This expression literally assumes $\omega \ll m_b$, $k_\perp \ll m_b$ and the radiation angle $\theta \simeq \frac{k_\parallel}{\omega} \ll 1$. More accurate expressions for arbitrary θ do not change the principal point here.

The radiation probability (1) underlies the conceptual problem: even if the gluon is very energetic by itself, $\omega \gg \mu$, the transverse momentum $k_\perp$ can be low, $|k_\perp| \lesssim \Lambda_{\rm QCD}$ if the gluon is highly collinear with $\theta \ll \frac{\mu}{\omega} < 1$. The 'perturbative' bremsstrahlung then

runs into the strong-coupling domain and cannot be unambiguously evaluated expanding in $\alpha_s(Q^2)$ with $Q^2 \gtrsim \mu^2$. This observation may cast doubts on calculability of even the 'robust' observables like the moments of the decay distributions.

A more careful analysis, however suggests that the primary OPE results for the (integer) moments relating them to the local heavy quark operators, remain unchanged, while the spectra themselves, in general, may possibly depend to some extent on new strong-coupling effects appearing in jet physics.

2   OPE for the decay distributions in $b \to q$

For the sake of simplicity we will phrase the subsequent discussion for the decay $b \to s + \gamma$ (or, generically, $Q \to q + \varphi$ with $m_q = 0$ and $m_\varphi = 0$, referring to the colorless φ as a photon whether or not it carries spin). We will also assume a much stronger hierarchy $m_b = m_Q \gg \mu_{\mathrm{hadr}}$ than exists in reality, so that we may discard inessential $1/m_b$ corrections ab initio.

The photon spectrum in $b \to s + \gamma$ is represented by the convolution of the soft 'primordial' distribution function $F(k_+)$ often known [2, 3] as describing "Fermi motion" [4, 5] of the b quark inside B meson, and of the 'hard' spectrum evolving from the bare two-body $\delta(E_\gamma - \frac{m_b}{2})$ due to the gluon bremsstrahlung:

$$\frac{\mathrm{d}\Gamma_{\mathrm{tot}}(E_\gamma)}{\mathrm{d}E_\gamma} = \int_{-\infty}^{\infty} \mathrm{d}k_+ \, F(k_+) \, \frac{\mathrm{d}\Gamma_{\mathrm{pert}}}{\mathrm{d}E}(E_\gamma - \tfrac{k_+}{2}) \; . \tag{2}$$

The support of $F(k_+)$ actually lies below $\overline{\Lambda} \simeq M_B - m_b(\mu)$; at large negative arguments with $|k_+| \gg \mu_{\mathrm{hadr}}$ the distribution function $F(k_+)$ must decrease exponentially.

Kinematically we have

$$E_\gamma = \frac{M_B^2 - M_X^2}{2M_B} \; , \tag{3}$$

where M_X^2 is the invariant hadronic mass squared in the final state (jet invariant mass, in the perturbative description). Instead of the photon spectrum we can, therefore speak of the distribution in M_X^2, or of the moments of the hadronic mass squared which has more universal application in jet physics. The typical nonperturbative (bound-state) domain in M_X^2 (referred to as 'window' in Ref. [3]) is $M_X^2 \lesssim \overline{\Lambda} m_b$, and larger M_X^2 emerges due to hard bremsstrahlung.

As schematically illustrated in Fig. 1, $F(k_+)$ describes the intrinsic properties of the decaying bound state. It is universal with respect to any concrete type of $Q \to q$ transition as long as the recoiling system is colorless and the energy of the light quark is large. All field modes included in it have wavelengths limited by μ. Therefore, it is completely independent of m_b once power corrections $\sim \mu_{\mathrm{hadr}}/m_b$ are neglected.

$F(k_+)$ is constrained by its (integer) moments like the usual DIS structure functions. The moments are given by the expectation values of the local heavy quark operators over
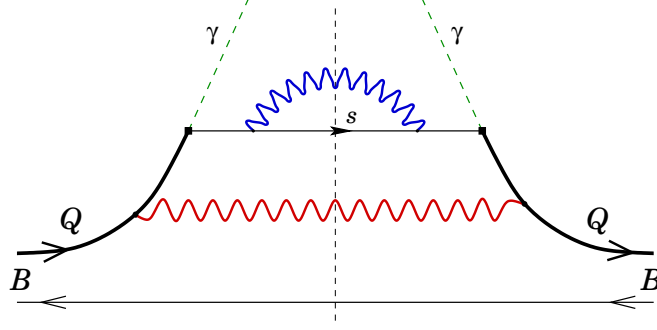
Figure 1: The corrections to the inclusive decay distribution in the OPE illustrated with the perturbative diagrams. The red gluon line shows the 'soft' modes with all the components of the four-momentum small. They are responsible for the 'Fermi motion' and are associated with the decaying bound state. The blue gluon line stands for harder gluons with $\omega > \mu$ attributed to Γ_{pert}, which nevertheless can be highly collinear and emitted with large coupling.

the initial hadron:

$$
F_0 = \int dk_+ \, F(k_+) = 1 \, , \qquad\qquad F_1 = \int dk_+ \, k_+ F(k_+) = 0 \, ,
$$

$$
F_2 = \int dk_+ \, k_+^2 \, F(k_+) = \frac{\mu_\pi^2}{3} \, , \qquad F_3 = \int dk_+ \, k_+^3 \, F(k_+) = -\frac{\rho_D^3}{3} \, ,
$$

$$
F_n = \int dk_+ \, k_+^n \, F(k_+) = \frac{1}{2M_{H_Q}} \langle H_Q | \bar{Q} i D_z (i D_0 - i D_z)^{n-2} i D_z Q | H_Q \rangle \, . \tag{4}
$$

In Eq. (2) $\frac{d\Gamma_{\text{pert}}}{dM_X^2}$ accounts for all other gluon modes. In practical terms, the renormalization scheme [6] is advantageous where, at the level of one gluon emissions the normalization procedure sets the separation based on the gluon energy ω. Then, for radiating a single gluon we have to perform the integration with only the lower cutoff on ω, for instance

$$
\frac{d\Gamma^{\text{pert}}}{dM_X^2} = \int \frac{d\omega}{\omega} \, \vartheta(\omega - \mu) \int \frac{dk_\perp^2}{k_\perp^2} \, C_F \frac{\alpha_s(k_\perp^2)}{\pi} \, \delta(M_X^2 - k_\perp^2 \frac{m_b}{2\omega}) \, . \tag{5}
$$

Here $m_b/2$ simply stands for the initial energy of the s quark jet E_{jet}. The kinematic enhancement factor $m_b/(2\omega)$ in M_X^2 brings in the contribution of large transverse distances $x_\perp > \mu_{\text{hadr}}^{-1}$ into the calculation of the perturbative spectrum in Eq. (5) once M_X^2 becomes smaller than $\frac{m_b}{2}\mu_{\text{hadr}}\frac{\mu_{\text{hadr}}}{\mu}$. The factor $\frac{\mu_{\text{hadr}}}{\mu}$ is smaller than unity, yet it represents a numeric rather than parametric suppression. Perturbation theory alone is unable to precisely calculate its decay spectrum closer to the endpoint than a fraction of the distribution function domain (window), $\mu_{\text{hadr}} m_b$. This fraction appears to depend on the normalization point. Consequently, the full point-to-point spectrum may not be completely expressed through the distribution function with the resolution arbitrarily higher than the size of the Fermi motion 'window'.

3

It is important that the type of the strong-coupling effects generated by the 'perturbative' jet function Eq. (5) is physically distinct from the Fermi motion. The latter is determined by the bound-state dynamics and depends on the B meson wavefunction at distances $\sim \mu_{\mathrm{hadr}}^{-1}$ from the heavy quark and its decay vertex. The jet hadronization, on the contrary, runs into the domain of the nonperturbative coupling only over the time interval $\sim \mu_{\mathrm{hadr}}^{-1}$ in the frame accompanying the final light quark, a much longer period $\tau_{\mathrm{jet}} \gtrsim \mu_{\mathrm{hadr}}^{-1} \sqrt{\frac{m_b}{\mu_{\mathrm{hadr}}}}$ due to the Lorentz slowdown. The hadronizing system travels over a distance $\gtrsim m_b^{\frac{1}{2}} \mu_{\mathrm{hadr}}^{-\frac{3}{2}}$ before that. The nonperturbative hadronization occurs far away in space from the decay point where the original B meson was located. This suggest that the considered nonperturbative jet effects are largely universal and are independent of the properties of the initial bound state. There are arguments that this separation is exponential in the ratio μ/μ_{hadr}.[1] We will return to this point later.

3 Moments of the distribution

The incalculable nonperturbative end point in $\frac{d\Gamma^{\mathrm{pert}}}{dM_X^2}$ and the associated indeterminacy of $\frac{d\Gamma_{\mathrm{tot}}}{dM_X^2}$ raise the concern of whether the moments of the total spectrum can still be obtained, in the presence of gluon bremsstrahlung, in terms of the local heavy quark expectation values applying only the truly short-distance corrections evaluated in perturbation theory over the small-coupling domain. At first glance, Eq. (5) mandates the presence of additional nonperturbative corrections from jet physics at small $k_\perp^2$, at least at the level $(m_b \mu_{\mathrm{hadr}})^n \cdot (\mu_{\mathrm{hadr}}/\mu)^n$ for the nth moment. In the zeroth moment of $\frac{d\Gamma^{\mathrm{pert}}}{dM_X^2}$, the total decay rate Γ_{pert} such effects cancel between the bremsstrahlung and the virtual corrections as required by the KLN theorem. The virtual effects are absent from the positive moments, and the simple-minded application of Eq. (5) yields non-vanishing nonperturbative contributions already to $\langle M_X^2 \rangle$ proportional to the first moment $\mathcal{A}_2 = -A_2$ of the effective coupling (the log-moment of the dispersion coupling $\alpha_{\mathrm{eff}}(s)$) [7] parameterizing power corrections in a number of observables in jet physics.

We find, however that such a conclusion would be incorrect. The nonperturbative jet effects, although possibly present in the differential distribution itself, cancel out in the integer moments. The bremsstrahlung corrections to the usual nonperturbative expressions for the moments in terms of the B-meson local heavy quark expectation values, calculated in perturbation theory, do not involve the running coupling at scales below μ.[2]

The most transparent way to illustrate physics behind the related 'conspiracy' in the strong-coupling jet effects uses the so-called dispersive approach to perturbation theory in Minkowski space scrutinized by Dokshitzer, Marchesini and Webber [8]; a recent brief summary for purely perturbative BLM-type applications can be found in Refs. [9]. The

---

[1]Yu. Dokshitzer, private communication.

[2]We do not consider the moment rank n as a large parameter and, therefore do not distinguish between, say $\alpha_s(\mu)$ and $\alpha_s(\mu/\sqrt{n})$.

method is based on the dispersion representation for the dressed gluon propagator

$$\frac{\alpha_s^E(Q^2)}{Q^2} = \pi \int \frac{d\lambda^2}{\lambda^2} \frac{\rho(\lambda^2)}{\lambda^2 + Q^2} , \qquad \rho(s) = -\frac{1}{\pi^2} \mathrm{Im}\, \alpha_s(-s) \tag{6}$$

via a fictitious gluon mass λ; Q^2 is the Euclidean virtuality and α_s^E denotes the standard Euclidean running coupling (the above expression for simplicity assumes no pole in $\alpha_s^E(Q^2)/Q^2$ at $Q^2 = 0$). This requires calculating the inclusive observable in question O to order α_s with an arbitrary gluon mass λ, $O^{\mathrm{born}} + C_F \frac{\alpha_s}{\pi} O_1(\lambda^2)$. The effect of the running α_s is then obtained by

$$O^{\mathrm{resum}} = O^{\mathrm{born}} + C_F \int \frac{d\lambda^2}{\lambda^2} \rho(\lambda^2)\, O_1(\lambda^2) \tag{7}$$

(the order-$\beta_0 \alpha_s^2$ version of this BLM improvement was formulated in Ref. [10]). In this context the OPE separates the effects into 'perturbative' and 'nonperturbative' according to their behavior in the deep Euclidean domain, not for the Minkowski objects. Therefore, one puts

$$\rho(\lambda^2) = \rho_{\mathrm{pert}}(\lambda^2) + \delta\rho(\lambda^2) \tag{8}$$

and it is assumed for the corresponding dispersion integrals, $\alpha_s^E(Q^2)$ and $\delta\alpha_s^E(Q^2)$, that the latter dies out fast at large $Q^2 > 0$.

Being interested in the effects from the strong-coupling regime we need to retain only the $\delta\alpha_s^E(Q^2)$, or $\delta\rho(\lambda^2)$ piece. For simplicity we will use the full coupling in the following equations, however, implying that the 'nonperturbative' part can be separately considered when required.

The standard bremsstrahlung probability within this framework takes the following form:

$$dW_{\mathrm{brem}} = C_F \int \frac{d\omega}{\omega} \int \frac{d\lambda^2}{\lambda^2} \rho(\lambda^2) \int \frac{dk_\perp^2}{k_\perp^2 + \lambda^2}\, dW_{\mathrm{born}} . \tag{9}$$

The integral over λ^2 is nothing but the dispersion representation for $\frac{\alpha_s(k_\perp^2)}{k_\perp^2}$:

$$\int \frac{d\lambda^2}{\lambda^2} \rho(\lambda^2) \frac{1}{k_\perp^2 + \lambda^2} = \frac{1}{\pi} \frac{\alpha_s(k_\perp^2)}{k_\perp^2} , \tag{10}$$

justifying the standard prescription Eq. (1).

As already mentioned, there is the KLN cancellation of soft gluons between the bremsstrahlung and the virtual corrections. The latter do not contribute to the higher moments, however we get generally the power-divergent integrals like $\int \frac{dk_\perp^2}{k_\perp^2}(k_\perp^2)^n \alpha_s(k_\perp^2)$. The effective $\alpha_s(k_\perp^2)$ then has to be accounted for with higher accuracy. To do this we write explicitly, e.g. for the first moment

$$\langle M_X^2 \rangle^{\mathrm{pert}} = C_F \int dM_X^2 \int \frac{d\omega}{\omega} \vartheta(\omega - \mu) \int \frac{d\lambda^2}{\lambda^2} \rho(\lambda^2) \int \frac{dk_\perp^2}{k_\perp^2 + \lambda^2}\, M_X^2 \delta(M_X^2 - (k_\perp^2 + \lambda^2)\tfrac{m_b}{2\omega}) . \tag{11}$$

5

Performing the integration over M_X^2 yields

$$\langle M_X^2 \rangle^{\text{pert}} = C_F \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega - \mu)\, \frac{m_b}{2\omega}\, \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2)\, \int \mathrm{d}k_\perp^2 \ . \tag{12}$$

The integral has split into the product of independent integrals over $k_\perp^2$ and over λ^2, which means that the effective coupling enters only at the high scale $\sim \sqrt{\mu\, m_b}$ or higher, cf. Eqs. (14), (16), determined by the effective ultraviolet cutoff in the integral. Formally, the absence of the nonperturbative contributions follows from

$$\pi \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \delta\rho(\lambda^2) = \lim_{Q^2 \to \infty} \delta\alpha_s^E(Q^2) = 0 \ . \tag{13}$$

The expression for the spectrum itself helps to interpret this general result:

$$\frac{\mathrm{d}\Gamma^{\text{pert}}}{\mathrm{d}M_X^2} = C_F \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega - \mu)\, \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2)\, \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2 + \lambda^2}\, \delta(M_X^2 - (k_\perp^2 + \lambda^2)\tfrac{m_b}{2\omega})$$
$$= \frac{C_F}{M_X^2} \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega - \mu)\, \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2)\, \vartheta(M_X^2 - \tfrac{m_b}{2\omega}\lambda^2) \ . \tag{14}$$

This differs from the expression

$$\frac{C_F}{M_X^2} \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega - \mu)\, \frac{\alpha_s^E(\tfrac{2\omega}{m_b}M_X^2)}{\pi} \tag{15}$$

which would literally follow from the prescription (5). The effective coupling in the spectrum $\tilde{\alpha}_s(\tfrac{2\omega}{m_b}M_X^2)$ differs from the usual Euclidean coupling $\alpha_s(\tfrac{2\omega}{m_b}M_X^2)$:

$$\tilde{\alpha}_s(Q^2) = \pi \int_0^{Q^2} \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \qquad \text{while} \qquad \alpha_s^E(Q^2) = \pi \int_0^\infty \frac{\mathrm{d}\lambda^2}{\lambda^2 + Q^2}\, \rho(\lambda^2) \tag{16}$$

The two couplings agree in the perturbative domain with 'log' accuracy where the momenta appear ordered in their scale. Yet they become different in the strong coupling regime. In particular, the moments of $\delta\alpha_s^E(Q^2)$ are nonzero and, for the simplest ansatsë [8] all have the same sign scaling as μ_{hadr}^{2n}. The integer moments of $\delta\tilde{\alpha}_s(Q^2)$, on the contrary, all vanish [11, 8] according to the generalization of the relation (13):

$$\pi \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \lambda^{2n}\, \delta\rho(\lambda^2) = (-1)^n \lim_{Q^2 \to \infty} Q^{2n}\, \delta\alpha_s^E(Q^2) \ . \tag{17}$$

While the general expression for the probability of the gluon bremsstrahlung, Eq. (1) remains valid, in a sense, even nonperturbatively, the actual physical effect differs from what would follow from Eq. (1) at the power level. The latter therefore has limited applicability. The reason is that at this level the one-particle massless-gluon description of the interaction becomes incompatible with running of α_s, a field-theory effect associated with a few particle states of somewhat different kinematics. Having the gluon split,

while not modifying the total rate, changes fine kinematic details thus reshuffling the distributions. This modifies the effective coupling for differential distributions, making it observable-dependent once the result is forcibly interpreted in terms of the massless one-gluon framework. Additionally, these effective couplings obey certain integral constraints which effectively recognize that the Euclidean image of the inclusive bremsstrahlung is a genuinely short-distance process.

The result of the calculations supports the interpretation that, in the hard collinear jet configurations the growth of α_s from the initial $\alpha_s(E_{\rm jet})$ to $\alpha_s(k_\perp)$ is an effect of the final-state interaction, viz. jet splitting. As such, it is not expected to affect fully inclusive truly short-distance characteristics like the total decay rate or recoil moments. We see that the long-time jet contributions disappear in just the integer moments of M_X^2, indicating that M_X^2 is the right kinematic variable.

4 Discussions

Jet hadronization effects may affect the hadronic mass distribution and, consequently, the point-to-point recoil spectrum in the decays like $B \to X_s + \gamma$ or $B \to X_u \, \ell\nu$. Yet it is shown that there remain no such nonperturbative strong-coupling–domain contributions to the moments. The perturbative corrections to the OPE relations for the moments come from the small coupling regime. Nonperturbatively the latter are shaped by the soft modes but not by the collinear modes.

There appears an interesting analogy between the strong-coupling effects in high-energy jets and local duality violation in the heavy quark decays specifically studied in Ref. [12] in the instanton vacuum ansatz. In both cases the nonperturbative effects were present and enhanced close to the end point in the differential distributions. Integrating them over the available kinematic domain might seem to only increase the effect. Yet for the right moments the integrated effect is always decreased being driven by the suppression in the corner of the underintegrated domain; it is minimal for totally inclusive rates. This similarity, perhaps has its roots in the general properties of the OPE, although the precise structure of the latter for jet physics still remains to be understood.

The identified property of the large-energy low-$k_\perp$ gluon radiation leading to the absence of the soft domain contribution from the integer moments, seems to have something in common with the observation by Beneke and Braun [13] of the vanishing of a certain class of the leading, linear in $\Lambda_{\rm QCD}/E$ corrections in the Drell-Yan production.

The literal incalculability of the spectrum itself in the large-m_Q limit near the end point may still seem unsatisfactory. It may be that there exists a further conspiracy in the small-$k_\perp$ jet splitting which eliminates this. A possible source for insights is to examine the formal μ-dependence of the incalculable piece. The spectrum must be μ-independent; at the same time, the kinematic M_X^2-domain affected by large-$x_\perp$ physics appears to depend on the lower gluon energy cutoff μ. The properties of the thus defined $F(k_+; \mu)$ have not been studied in detail, however, which precludes us from definite conclusions. The problem deserves further dedicated analysis.

7

It is conceivable that such nontrivial effects at the interface of the gluon bremsstrahlung and of the genuine nonperturbative corrections in the point-to-point spectrum are related to the specific way one defines the primordial distribution function assuming the Wilsonian cutoff on all the components of the momentum of the soft modes. Such a definition is motivated by the physics of the heavy quark bound state, but it looks foreign to the light cone approach. In the infinite momentum frame accompanying the jet the light-cone $k_\perp \leftrightarrow x_\perp$ and $E_+ \sim M_X^2$ are natural variables. Using them for the separation of mass scales would allow, however large wave vectors $k_\parallel$ which are foreign to the bound-state dynamics.

Including the low-$k_\perp$ high-energy jet hadronization effects into the definition of an effective distribution function $F(k_+)$ may seem to be a practical way to get around the incalculability of the perturbative spectrum. This idea might seem to be supported by the mentioned universality of the jet hadronization. The advantage is doubtful, however. Long-wavelength 'Fermi motion' and highly collinear jet interactions seem to describe quite distinct physics, as mentioned in Sect. 2. Likewise the jet effects have little in common with the B meson expectation values of the local heavy quark operators determining the moments of $F(k_+)$, or with the heavy quark bound-state dynamics in general. Moreover, they would a priori bring in the intrinsic (logarithmic) dependence of $F(k_+)$ on the high scale m_Q, from which the conventional definition is free.

The strong-coupling regime effects addressed above are not specific to the weak decays of heavy flavors, but are common to general high-energy light-like jet processes in hadronic physics. We specifically phrased the discussion in the context of the inclusive heavy quark decays since here it can be put in a more rigorous context of the OPE, absorbing the soft modes into the heavy quark distribution function $F(k_+; \mu)$.

In our analysis no indications were found towards the specific additional intermediate momentum scale $\sim \Lambda_{\mathrm{QCD}} m_b$ for the inclusive decay distributions. The bremsstrahlung integration runs over all virtualities: the concrete domain contributing is simply determined by the energy of the radiated gluon, ranging from small virtualities up to m_b^2.

References

[1] B. Aubert et al., BABAR Collaboration, *Phys. Rev. Lett.* **93** (2004) 011803.

[2] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Phys. Rev. Lett.* **71** (1993) 496.

[3] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Int. Journ. Mod. Phys.* **A9** (1994) 2467.

[4] A. Ali, E. Pietarinen *Nucl. Phys.* **B154** (1979) 519.

[5] G. Altarelli et al., *Nucl. Phys.* **B208** (1982) 365.

[6] N. Uraltsev, *Nucl. Phys.* **B491** (1997) 303; *Phys. Lett.* **B545** (2002) 337; I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Phys. Rev.* **D56** (1997) 4017.

[7] M. Dasgupta, G.E. Smye and B.R. Webber, *JHEP* **9804** (1998) 017; Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, *ibid.* **9907** (1999) 012.

[8] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, *Nucl. Phys.* **B469** (1996) 93.

[9] N. Uraltsev, *Int. Journ. Mod. Phys. Lett.* **A17** (2002) 2317; D. Benson, I. Bigi, Th. Mannel and N. Uraltsev, *Nucl. Phys.* **B665** (2003) 367.

[10] B.H. Smith and M.B. Voloshin, *Phys. Lett.* **B340** (1994) 176.

[11] R.D. Dikeman, M. Shifman and N.G. Uraltsev, *Int. Journ. Mod. Phys.* **A11** (1996) 571.

[12] B. Chibisov, R. Dikeman, M. Shifman and N. Uraltsev, *Int. J. Mod. Phys.* **A12** (1997) 2075.

[13] M. Beneke and V.M. Braun,*Nucl. Phys.* **B454** (1995) 253.